

Revisiting the

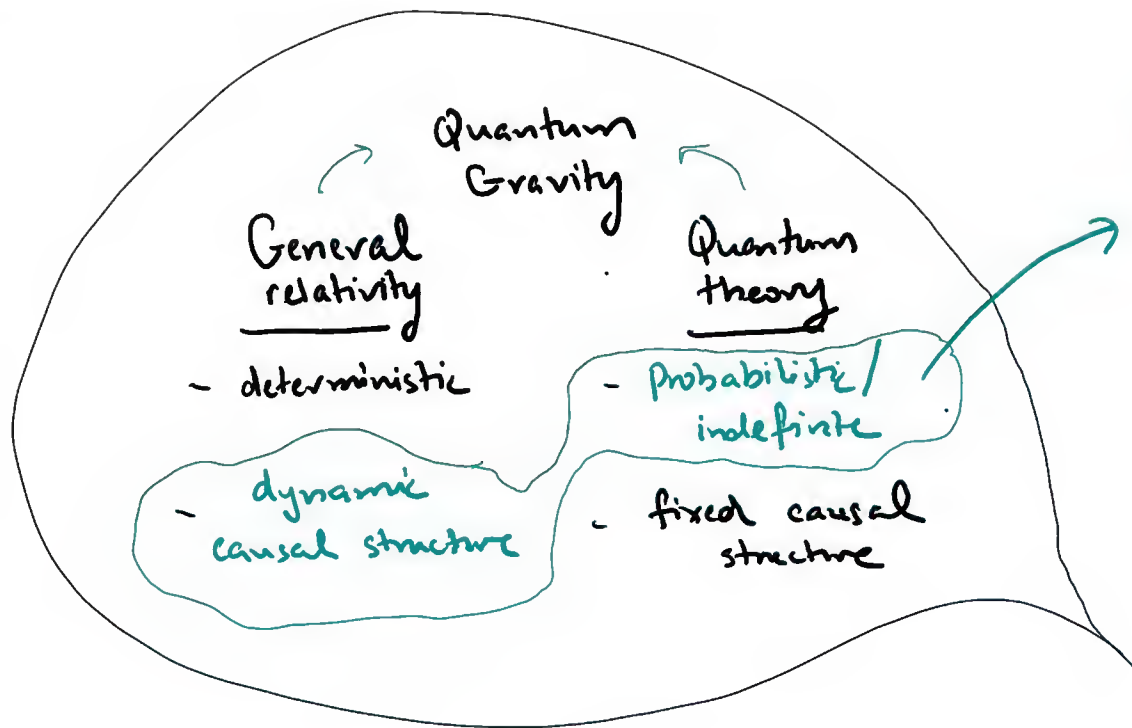
Causabid Framework

Nitica Sakharwade

Perimeter Institute



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0509120



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Operational Methodology

F : Procedure Set

Y : Outcome Set

R_i : Elementary Region

$F_{R_i} := F \cap R_i$: Procedure in R_i

$Y_{R_i} := Y \cap R_i$: Outcome in R_i



"Thinking inside the box"

Operational Methodology

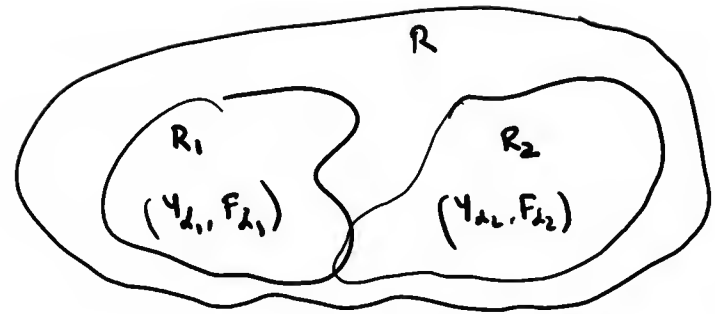
F : Procedure Set

Y : Outcome Set

R_i : Elementary Regions^{*}, $R = \bigcup_i R_i$

$F_{R_i} := F \cap R_i$: Procedure in R_i

$Y_{R_i} := Y \cap R_i$: Outcome in R_i



We are interested
in the probability

$$P(Y_{\alpha_1} | Y_{\alpha_2}, F_{\alpha_1}, F_{\alpha_2})$$

$$\forall (Y_{\alpha_i}, F_{\alpha_i}) \text{ in } R_i \text{ and } \forall R_i \in R$$
$$\alpha_i = 1, 2, \dots$$



"Thinking inside the box"

Redundancy & Compression

A physical theory will correlate some of these probabilities to give a smaller set

This is **PHYSICAL COMPRESSION**

In the causal framework,
there are **three** levels of compression

1st level

Consider only a single region R_1

$$P(Y_{\alpha_1} | F_{\alpha_1}) \equiv P_{\alpha_1}$$

$$P_{\alpha_1} = r \cdot p$$

$$\text{where } p = \begin{pmatrix} p_1 \\ \vdots \\ p_{\alpha_1} \\ \vdots \end{pmatrix}$$

$$\text{and } r = (0 \dots 0 \dots 1 \dots 0 \dots)$$

$$\text{or } r_i = \delta_{i\alpha_1}$$

1st level

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But there are infinite possible measurement/procedure!

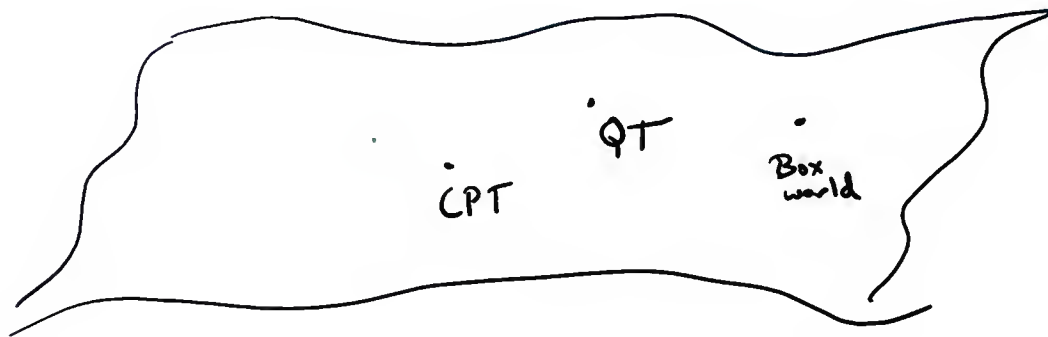
We can choose a fiducial set Ω_1 to fully characterise a state

$$P_{\alpha_1} = \lambda_{\alpha_1}^{l_1} P_{\alpha_1} \quad l_1 \in \Omega_1$$

$$\alpha_1 \rightarrow l_1 \in \Omega_1$$

1st level

This is nothing but the compression used to define the landscape of **Generalised Probability theory** used in reconstruction of QT using new axioms.



Generalised Prob Theories (Non-locality)

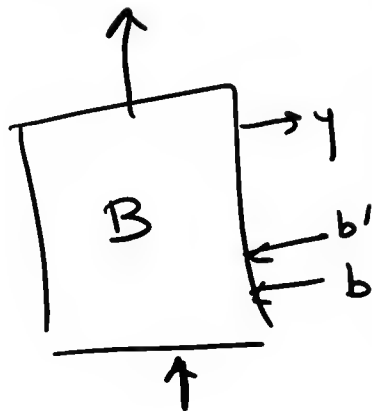
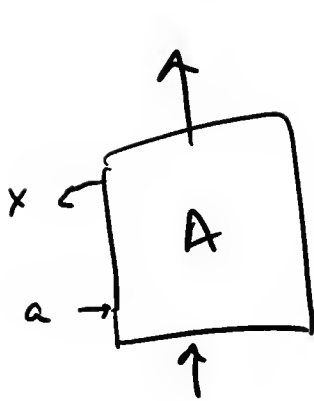
	local	P_{success} (non-local game)	
CPT	✓	0.75	(local bound)
QT	x	~0.85	(Tsirelson's bound)
PR boxes	x	1.00	(logical bound)



What does the violation
of causal inequalities
mean?

Causal Inequality (Oreshkov, Costa, Brukner)

Guess your neighbor's input?



$$P_{\text{success}} = \frac{1}{2} \left(P(x=b \mid b'=1) + P(y=a \mid b'=0) \right)$$

if $P_{\text{success}} > 3/4 \rightarrow \text{violation!}$

Landscape

DCS
+ QC-CC
+ QC-QC

QT + ICs

?

P_{success} (GYNI Game)

0.75

?

1.00

(logical bound)



Can there be a **landscape** of
probabilistic theories with
indefinite causal structure?

(causaloid ?)

2nd level

(or more)

Consider two regions R_1, R_2

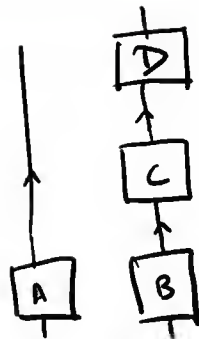
$$P_{\lambda_1, \lambda_2} = \underbrace{\Lambda_{l_1}^{\lambda_1} \Lambda_{l_2}^{\lambda_2}}_{1^{st}} \underbrace{\Lambda_{u_1 u_2}^{l_1 l_2}}_{2^{nd}} \cdot P_{u_1 u_2}$$

$$l_1, l_2 \dots \rightarrow u_1 u_2 \dots$$

$$u_1 u_2 \in \Omega_{12} \in \Omega_1 \times \Omega_2$$

$$l_1, l_2 \in \Omega_1 \times \Omega_2$$

TYPES



① $A \times B$

② CB

causally adjacent

③ $[D ? B]$

supermap/comb

Causally adjacent type
show compression

3rd level

Further compress!

$$\bigwedge_{\alpha i}^{l_i} \forall R_i \quad (1^{\text{st}} \text{ level})$$

$$\bigwedge_{k_i l_j \dots}^{k_i k_j \dots} \forall R_i R_j \dots \quad (2^{\text{nd}} \text{ level})$$

$$k_i l_j \dots \in \Omega_{ij} \dots$$

$$l_i l_j \dots \in \Omega_i \times \Omega_j \times \dots$$

Ansatz (if no loops)

$$\Omega_{1\dots n} = 0 \left(\begin{array}{c} n_2 \\ \prod_{i,j=3} \Omega_{ij} \\ \hline \left(\prod_i \Omega_i \right)^{x^{n-2}} \end{array} \right)$$

reorder

Thoughts?

Thank You!